A SIMPLE DETERMINATION OF THE THERMODYNAMICAL CHARACTERISTICS OF THE WEAKLY CHARGED, VERY THIN BLACK RING

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Abstract

In our previous work we suggested a very simple, approximate formalism for description of some basic (especially thermodynamical) characteristics of a non-charged, rotating, very thin black ring. Here, in our new work, generalizing our previous results, we suggest a very simple, approximate description of some basic (especially thermodynamical) characteristics of a weakly charged, rotating, very thin black ring. (Our formalism is not theoretically dubious, since, at it is not hard to see, it can represent an extreme simplification of a more accurate, e.g. Copeland-Lahiri, string formalism for the black hole description.) Even if suggested formalism is, generally speaking, phenomenological and rough, obtained final results, unexpectedly, are non-trivial. Concretely, given formalism reproduces exactly Bekenstein-Hawking entropy, Bekenstein quantization of the entropy or horizon area and Hawking temperature of a weakly charged, rotating, very thin black ring originally obtained earlier using more accurate analysis by Emparan, Aestefanesei, Radu etc. (Conceptually it is similar to situation in Bohr's atomic model where energy levels are determined practically exactly even if electron motion is described roughly.) Our formalism is physically based on the assumption that circumference of the horizon tube holds the natural (integer) number of corresponding reduced Compton's wave length. (It is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation.) Also, we use, mathematically, practically only simple algebraic equations.

In our previous work [1] we suggested a very simple, approximate formalism for description of some basic (especially thermodynamical) characteristics of a non-charged, rotating, very thin black ring. Here, in our new work, generalizing our previous results, we suggest a very simple, approximate description of some basic (especially thermodynamical) characteristics of a weakly charged, rotating, very thin black ring. (Our formalism is not theoretically dubious, since, at it is not hard to see, it can represent an extreme simplification of a more accurate, e.g. Copeland-Lahiri [2], string formalism for the black hole description.) Even if suggested formalism is, generally

speaking, phenomenological and rough, obtained final results, unexpectedly, are non-trivial. Concretely, given formalism reproduces exactly Bekenstein-Hawking entropy, Bekenstein quantization of the entropy or horizon area and Hawking temperature of a weakly charged, rotating, very thin black ring originally obtained earlier using more accurate analysis by Emparan [3], Aestefanesei, Radu [4] etc. (Conceptually it is similar to situation in Bohr's atomic model where energy levels are determined practically exactly even if electron motion is described roughly.) Our formalism is physically based on the assumption that circumference of the horizon tube holds the natural (integer) number of corresponding reduced Compton's wave length. (It is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation.) Also, we use, mathematically, practically only simple algebraic equations (by determination of Hawking temperature we use additionally only simple differentiation of Smarr relation).

As it is well-known [3], [5], [6], horizon of a rotating, very thin black ring can be approximated by a torus with great radius (i.e. distance from the center of the tube to the center of the torus)

$$R_1 = R(\frac{\lambda}{\nu})^{\frac{1}{2}} \tag{1}$$

and small radius (i.e. radius of the torus tube)

$$R_2 = R\nu \tag{2}$$

, where R, λ and ν represent corresponding variables so that, for rotating, very thin black ring, it is satisfied

$$0 < \nu < \lambda \ll 1. \tag{3}$$

Further, it is well-known [3], [4], that a weakly charged, rotating very thin black ring holds additional two variables, N (corresponding to dilaton coupling), and, μ (corresponding to local charge Q) that satisfy conditions

$$0 < N \leq 3 \tag{4}$$

$$0 < \ \ll 1. \tag{5}$$

Finally, it is well-known too [3], [4] that weakly charged, rotating, very thin black ring holds (in the natural units system $\hbar = G = c = k = 1$) the following mass - M, angular momentum - J, angular velocity - Ω, horizon area - A, entropy - S, temperature - T, local (electric) charge - Q, and (electric) potential - F

$$M = \frac{3\pi}{4}R^2(\lambda + \frac{N\mu}{3}) = \frac{3\pi}{4}R_1 R_2 (\frac{\lambda}{\nu})^{\frac{1}{2}}(1 + \frac{N\mu}{3\lambda}) \tag{6}$$

$$J = \frac{\pi R^3}{2}\lambda(1 - \frac{\nu}{\lambda})^{\frac{1}{2}} = \frac{\pi}{2}R_1^2 R_2 (1 - \frac{\nu}{\lambda})^{\frac{1}{2}} \tag{7}$$

$$\Omega = \frac{1}{R}(1 - \frac{\nu}{\lambda})^{\frac{1}{2}} = \frac{1}{R_1}(\frac{\lambda}{\nu} - 1)^{\frac{1}{2}} \tag{8}$$

$$A = 8\pi^2 R^3 \nu^{\frac{3-N}{2}} \lambda^{\frac{1}{2}} (\mu + \nu)^{\frac{N}{2}} = 8\pi^2 R_2^2 R_1 \nu^{\frac{-N}{2}} (\mu + \nu)^{\frac{N}{2}} \tag{9}$$

$$S = \frac{A}{4} = 2\pi^2 R_2^2 R_1 \nu^{\frac{-N}{2}} (\mu + \nu)^{\frac{N}{2}} \tag{10}$$

$$T = \frac{1}{4\pi R}\nu^{\frac{N-1}{2}} \lambda^{-\frac{1}{2}} (\mu + \nu)^{\frac{-N}{2}} = \frac{1}{4\pi R_2}(\frac{\nu}{\lambda})^{\frac{1}{2}} \nu^{\frac{N}{2}} (\mu + \nu)^{\frac{-N}{2}} \tag{11}$$

$$Q = RN^{\frac{1}{2}}\mu^{\frac{1}{2}} = R_1(\frac{\nu}{\lambda})^{\frac{1}{2}}N^{\frac{1}{2}}\mu^{\frac{1}{2}} \tag{12}$$

$$\Phi = \frac{\pi}{2}RN^{\frac{1}{2}}\mu^{\frac{1}{2}}(\mu+\nu)^{\frac{-1}{2}} = \frac{\pi}{2}R_2N^{\frac{1}{2}}\mu^{\frac{1}{2}}(\mu+\nu)^{\frac{-1}{2}}\nu^{-1}. \tag{13}$$

Now, as well as it has been done in our previous work [1], *suppose (postulate)* the following expression

$$m_{+n}R_2 = \frac{n}{2\pi}, \qquad \text{for} \qquad n = 1, 2, ... \tag{14}$$

where m_{+n} for $n = 1, 2,$ represent masses of some small quantum system captured at given weakly charged, rotating, very thin black ring horizon. It implies

$$2\pi R_2 = \frac{n}{m_{+n}} = n\lambda_{r+n}, \qquad \text{for} \qquad n = 1, 2, ... \tag{15}$$

Here, obviously, $2\pi R_2$ represents the circumference of small circle at given weakly charged, rotating, very thin black ring horizon, while

$$\lambda_{r+n} = \frac{1}{m_{+n}} \tag{16}$$

represents the n-th reduced Compton wavelength of a quantum system with mass m_{+n} captured at weakly charged, very thin black ring horizon for $n = 1, 2,$.

Expression (15) simply means that *circumference of given small circle (tube) at given weakly charged, rotating, very thin black ring horizon holds exactly n corresponding n-th reduced Compton wave lengths of mentioned small quantum system for* $n = 1, 2,$. (Obviously, it is conceptually similar to well-known Bohr's quantization postulate interpreted by de Broglie relation (according to which circumference of n-th electron circular orbit contains exactly n corresponding n-th de Broglie wave lengths, for $n = 1, 2,$).

According to (14) it follows

$$m_{+n} = \frac{n}{2\pi R_2} = nm_{+1}, \qquad \text{for} \qquad n = 1, 2, ... \tag{17}$$

where

$$m_{+1} = \frac{1}{2\pi R_2}. \tag{18}$$

Now, analogously to procedure in [1], *suppose (postulate) that given weakly charged, rotating, very thin black ring entropy is proportional to quotient of given weakly charged, rotating, very thin black ring mass M and minimal mass of mentioned small quantum system m_{+1}.* More precisely, suppose (postulate)

$$S = \gamma\frac{M}{m_{+1}} = \gamma(\frac{3\pi^2}{2})R_1R_2^2(\frac{\lambda}{\nu})^{\frac{1}{2}}(1+\frac{N\mu}{3\lambda}) \tag{19}$$

where γ represents some correction factor, i.e. parameter that can be determined by comparison of (19) and (10). It yields

$$\gamma = \frac{4}{3}(\frac{\nu}{\lambda})^{\frac{1}{2}}(\mu+\nu)^{\frac{N}{2}}\nu^{\frac{-N}{2}}(1+\frac{N\mu}{3\lambda})^{-1}. \tag{20}$$

Also, we shall define normalized mass of given weakly charged, rotating, very thin black ring

$$\tilde{M} = \gamma M = \pi R_1R_2(\mu+\nu)^{\frac{N}{2}}\nu^{\frac{-N}{2}}. \tag{21}$$

Further, we shall differentiate (9), (10) supposing approximately that R_2 represents unique variable while R_1, ν, λ, μ and N can be considered as constant parameters. It yields

$$dA = 16\pi^2 R_1 R_2 (\mu+\nu)^{\frac{N}{2}} \nu^{\frac{-N}{2}} dR_2 \tag{22}$$

$$dS = 4\pi^2 R_1 R_2 (\mu+\nu)^{\frac{N}{2}} \nu^{\frac{-N}{2}} dR_2. \tag{23}$$

Since, according to (21),

$$d\tilde{M} = \pi R_1 (\mu+\nu)^{\frac{N}{2}} \nu^{\frac{-N}{2}} dR_2 \tag{24}$$

that yields

$$dR_2 = \frac{d\tilde{M}}{\pi R_1 (\mu+\nu)^{\frac{N}{2}} \nu^{\frac{-N}{2}}} \tag{25}$$

, (22), (23) turn out in

$$dA = 16\pi R_2 d\tilde{M} \tag{26}$$

$$dS = 4p\pi R_2 d\tilde{M}. \tag{27}$$

Given expressions can be approximated by the following finite difference forms

$$\Delta A = 16\pi R_2 \Delta\tilde{M} \qquad \text{for} \quad \Delta\tilde{M} \ll \tilde{M} \tag{28}$$

$$\Delta S = 4\pi R_2 \Delta\tilde{M} \qquad \text{for} \quad \Delta\tilde{M} \ll \tilde{M} \tag{29}$$

Further, we shall assume

$$\Delta\tilde{M} = n m_{+1} \qquad \text{for} \qquad n = 1, 2, ... \tag{30}$$

Introduction of (30) in (28), (29), according to (18), yields

$$\Delta A = 8n = 2n(2)^2 \qquad \text{for} \qquad n = 1, 2, ... \tag{31}$$

$$\Delta S = 2n \qquad \text{for} \qquad n = 1, 2, ... \tag{32}$$

that exactly represent Bekenstein quantization of the black hole horizon surface (where $(2)^2$ represents the surface of the quadrate whose side length represents twice Planck length, i.e. 1) and entropy.

Now, we shall determine Hawking temperature of given weakly charged, rotating, very thin black ring in our approximation in the following way. We shall start from the first thermodynamical law for charged black rings

$$dM = TdS + \Omega dJ + Qd\Phi. \tag{33}$$

Further, we shall introduce an approximation, opposite to approximation introduced previously by deduction of Bekenstein entropy or horizon area quantization. Namely, we shall again suppose approximately that in (33) known functions M (6), J (7), Ω (8), S (10), Q (12) and Φ (13) as well as unknown function T that will be determined by (33) represent functions of only one variable R_2 while R_1, ν, λ, μ and N can be considered as constant parameters. Secondly, it can be supposed that under first supposition expression (33) cannot be exactly conserved so that given expression within given approximation must be corrected by two additional correction factors α and β in the following way

$$\alpha dM = TdS + \Omega dJ + \beta Qd\Phi. \tag{34}$$

Given correction factors α and β will be determined by comparison of the obtained value of T with exact expression (11).

Application of the introduced approximation rule and (6)-(10), (12), (13) on (34) yields

$$\alpha\frac{3\pi}{4}R_1(\frac{\lambda}{\nu})^{\frac{1}{2}}(1+\frac{N\mu}{3\lambda})dR_2 = \tag{35}$$

$$T4\pi^2 R_1 R_2 \nu^{\frac{-N}{2}}(\mu+\nu)^{\frac{N}{2}}dR_1 + \frac{\nu}{R_2}(1-\frac{\nu}{\lambda})^{\frac{1}{2}}\pi R_1 R_2 \nu^{-1}(\frac{\lambda}{\nu}-1)^{\frac{1}{2}}dR_2$$

$$+\beta R_2\frac{1}{\nu}N^{\frac{1}{2}}\mu^{\frac{1}{2}}(\mu+\nu)^{\frac{1}{2}}\frac{\pi}{2}(\frac{\nu}{\lambda})^{\frac{1}{2}}N^{\frac{1}{2}}\mu^{\frac{1}{2}}(\mu+\nu)^{\frac{-1}{2}}dR_1$$

which, after simple transformations, yields

$$T = \frac{1}{4\pi R_2}(\frac{\nu}{\lambda})^{\frac{1}{2}}\nu^{\frac{N}{2}}(\mu+\nu)^{\frac{-N}{2}}(\frac{3\alpha}{4}\frac{\lambda}{\nu}+\frac{\alpha}{4}\frac{N\mu}{\nu}-\frac{1}{2}\frac{\lambda}{\nu}+\frac{1}{2}-\frac{\beta}{2}\frac{N\mu}{\nu}). \tag{36}$$

Now, comparison of (36) and (11) implies

$$\alpha\frac{3}{4}\frac{\lambda}{\nu}-\frac{1}{2}\frac{\lambda}{\nu}=0 \tag{37}$$

$$(\frac{\alpha}{4})\frac{N\mu}{\nu}-(\frac{\beta}{2})\frac{N\mu}{\nu}=0 \tag{38}$$

that implies

$$\alpha = \frac{2}{3} \tag{39}$$

$$\beta = \frac{1}{3}. \tag{40}$$

It, on the one hand, means that (36) turns out in

$$T = \frac{1}{2}\frac{1}{4\pi R_2}(\frac{\nu}{\lambda})^{\frac{1}{2}}\nu^{\frac{N}{2}}(\mu+\nu)^{\frac{-N}{2}} \tag{41}$$

that represents one half of the exact Hawking temperature (11). It can be considered as a satisfactory result, even if there is no complete equivalence between exact, (11), and approximate, (41), term. On the other hand, introduction of the correction factors (39), (40), in the corrected first thermodynamical law (34) yields

$$\frac{2}{3}dM = TdS + \Omega dJ + \frac{1}{3}Qd\Phi \tag{42}$$

that, obviously, corresponds to Smarr relation [2], [3] exactly satisfied for black rings (including weakly charged, rotating, very thin black ring too)

$$\frac{2}{3}M = TS + \Omega J + \frac{1}{3}Q\Phi. \tag{43}$$

It, practically, implies that correction coefficients α and β can be determined by Smarr relation. In other word, within our approximative method, Hawking temperature can be determined exactly, starting from Smarr relation instead of first thermodynamical law.

In this way we have reproduced, i.e. determined simply, approximately three most important thermodynamical characteristics of a weakly charged, rotating, very thin black ring: Bekenstein-Hawking entropy, Bekenstein quantization of the horizon area or entropy, and, Hawking temperature . It can be observed, roughly speaking, that all these characteristics are caused by standing (reduced Compton) waves at small circles (torus tube circles) at horizon area only. In other words, within given approximation as well as within a more accurate analysis, thermodynamical characteristics of given rotating, very thin black ring are practically independent of the great circle of the torus.

In conclusion it can be shortly repeated and pointed out the following. In this work we suggested a simple, approximate formalism for description of some basic (especially thermodynamical) characteristics of a weakly charged, rotating, very thin black ring. (In fact, our formalism is not theoretically dubious, since, at it is not hard to see, it can correspond to an extreme simplification of a more accurate, Copeland-Lahiri string formalism for the black hole description.) Even if suggested formalism is, generally speaking, phenomenological and rough, obtained final results, unexpectedly, are non-trivial. Concretely, given formalism reproduces exactly Bekenstein-Hawking entropy, Bekenstein quantization of the entropy or horizon area and Hawking temperature of a rotating, very thin black ring obtained earlier using more accurate analysis (Emparan, Aestefanesei, Radu etc.). Our formalism is physically based on the assumption that circumference of the horizon tube holds the natural (integer) number of corresponding reduced Compton's wave length. (It is conceptually similar to Bohr's quantization postulate in Bohr's atomic model interpreted by de Broglie relation.) Also, we use, mathematically, practically only simple algebraic equations.

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